EFFECT OF CENTRIPETAL RADIAL BLOWING ON FLOW AND HEAT TRANSFER IN THE VICINITY OF A SCREENED DISK

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An investigation is made of the effect on the flow and heat transfer of blowing in a direction from the periphery to the center in the space between a rotating disk and a fixed screen, by means of a finite-difference computer solution of the Navier-Stokes and energy equations.

The questions examined in this article arose in the first instance from problems associated with construction of turbines. Radial flow in a direction from the periphery to the center in the gap between the disk of a turbine and the screen always occurs if the necessary pressure drop is present. The intensity of this flow depends also on the characteristics and the state of compression. Centripetal radial flows have an influence, in particular, on the moment of resistance of the rotating disk [1]. In radial gas turbines centripetal blowing of cold air is used to protect the metal disk from the action of the high temperature working gas [2]. In spite of the importance of this question, only isolated attempts [2, 3] have been made to give a theoretical solution of the problem. A sufficiently accurate solution may be obtained only by means of a computer. Some results of calculations are presented here for a laminar flow regime.

We note that assuming the relative gap s/r_0 between the disk and the screen (Fig. 1) to be small, and in the absence of reverse flow relative to the main mass flow, the equations of motion and energy may be reduced to a simplified form [4]:

$$v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\varphi^2}{r} = -\frac{1}{\rho}\frac{dp}{dr} + \nu \frac{\partial^2 v_r}{\partial z^2},$$

$$v_r \frac{\partial v_\varphi}{\partial r} + v_z \frac{\partial v_\varphi}{\partial z} + \frac{v_r v_\varphi}{r} = \nu \frac{\partial^2 v_\varphi}{\partial z^2},$$

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0,$$

$$\rho c_p \left(v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} \right) = \lambda \frac{\partial^2 T}{\partial z^2}. \qquad (1)$$

Here the only viscous terms retained in the equation are those containing derivatives with respect to the normal z to the disk surface, while the pressure becomes a function only of the radius, $p = p(r)$.

Solution of the system (1) of nonlinear equations of parabolic type with appropriate boundary conditions may be performed numerically with the aid of a two-layer, implicit, six-point scheme. This has been discussed in detail in previous papers [4, 5], where a description was also given of a program compiled by Serazetdinov and Ponomareva. The present calculations, in particular, were carried out with this program.

A series of calculations was performed with mesh steps of $\Delta r = 0.005 r_0$, $\Delta z = 0.025 s$. The distribution of components of the vector velocity and of the temperature at the initial peripheral radius r_0 were chosen as constant throughout the gap, apart from narrow zones at the wall:

$$v_\varphi \approx 0, \quad v_r \approx v_{r_0}, \quad T \approx T_s. \qquad (2)$$

A calculation was also made with a linear distribution v_φ. The assumption was made of constant temperature difference between the disk and screen, $T_d - T_s = \text{const}$. In this event the main dimensionless parameters of the problem become: rotation Reynolds number $Re = s^2\omega/\nu$, referred to radial velocity at the initial section $\varphi = v_{r_0} : r_0\omega$, and the Prandtl number $Pr = \mu c_p/\lambda$.

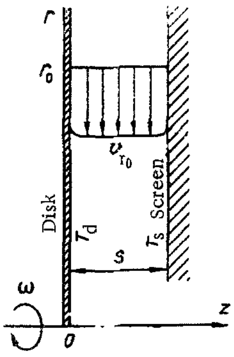

Fig. 1. Schematic of the problem.

The development of the flow in its progress (Fig. 2) indicates that the radial velocities increase almost in inverse proportion with decrease of r, in such a way that in the core of the flow there is only a negligible variation in $v_r r$ associated with "displacement" of the medium from the boundary layers at the walls. For a symmetric profile $v_r \approx v_{r_0}$ in the initial section, and $Re = -\varphi = 10$, the symmetry of the radial velocity profile is not disturbed. It is characteristic that even the flow swirl v_φ increases almost inversely with r, especially in the flow core: this is seen particularly clearly in the case when the distribution of v_φ is given as being linear over the gap in the initial section, $v_\varphi : r\omega = 1 - z/s$. Since $v_\varphi = r\omega$ on the disk, for this reason the circular component of friction stress of the disk, $\tau_\varphi = \mu(dv_\varphi/dz)_0$, changes sign along the radius. Therefore, if angular momentum is transferred from the disk to the flow in the initial section, then a transfer of angular momentum from the flow to the disk occurs at smaller values of r, and the "turbine" regime arises. This occurs particularly rapidly for the case when the swirl is nonzero at the initial section (see Fig. 3). This is the principle, as is well known, used in constructing the "friction" disk turbine, described

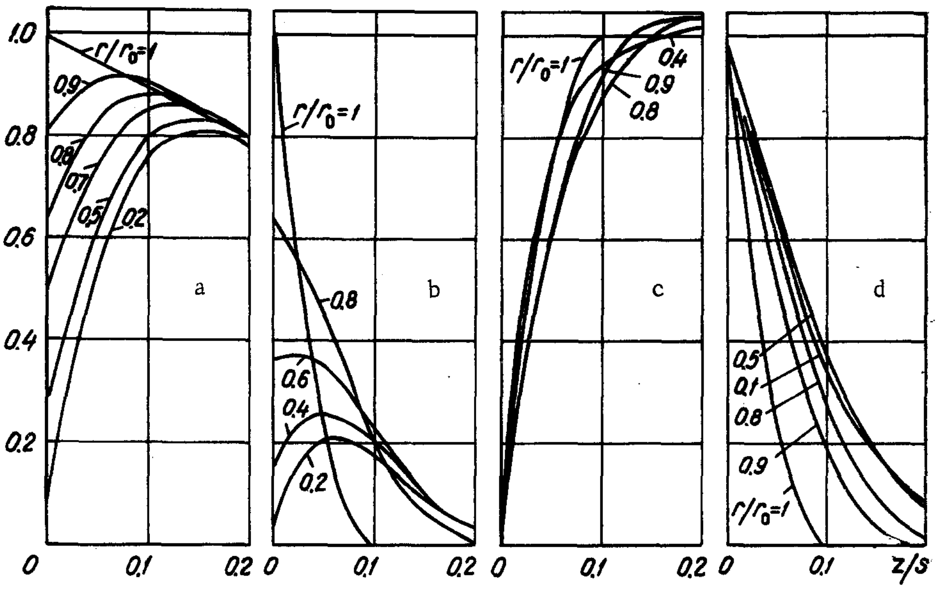

Fig. 2. Development of the flow in the gap along a radius for Re = 10, $\varphi = -10$, Pr = 0.7: a) flow swirl for a linear distribution in the initial section ($r = r_0$); b) swirl for $v_\varphi|_{r_0} \approx 0$ (in a and b the ordinate is $v_\varphi r/r_0^2\omega$); c) distribution of radial velocities (the ordinate is $v_r r/v_{r_0} r_0$); d) temperature distribution (the ordinate is $(T - T_s)/(T_d - T_s)$).

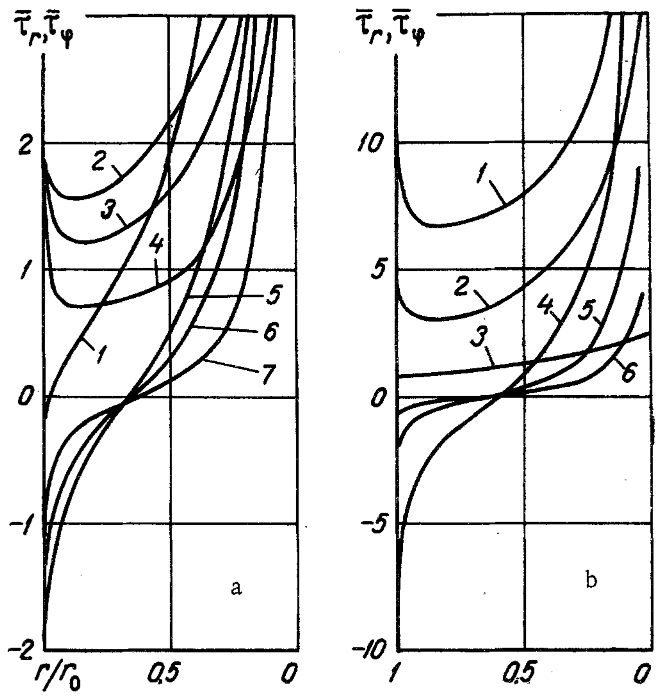

Fig. 3. Distribution along a radius of the circular and radial friction stress components on a rotating disk, with variation of mass flow (a) and of Reynolds number (b): a) for Re = 10: 1—the value of $\bar{\tau}_\varphi$ for a linear distribution v_φ in the initial section; 2, 3, 4—$\bar{\tau}_r$ for φ of 10, 5, 1, respectively; 5, 6, 7—$\bar{\tau}_\varphi$ for φ of 10, 5, 1, respectively; b) with φ = −5: 1, 2, 3—values of $\bar{\tau}_r$, respectively, with Re equal to 1, 10, 50; 4, 5, 6—$\bar{\tau}_\varphi$, respectively, for Re equal to 1, 10, 50.

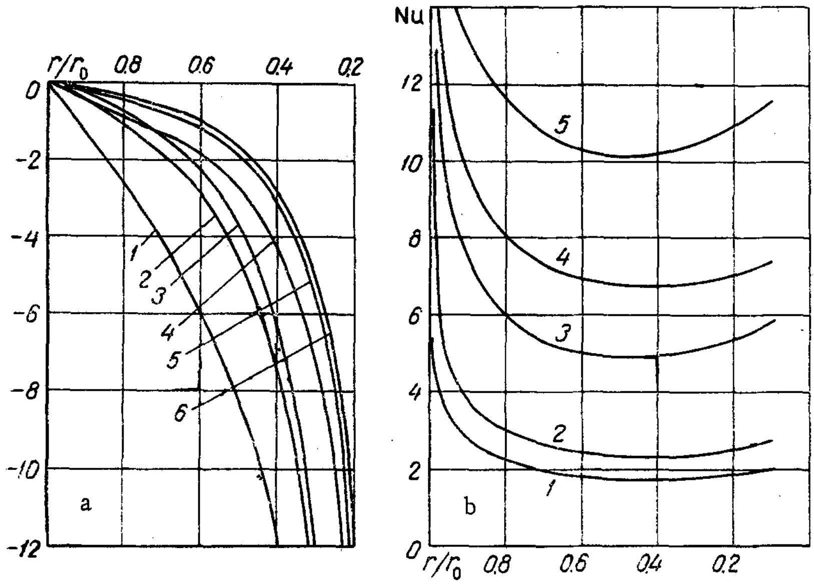

Fig. 4. Distribution along a radius of the pressure in the gap (a) and on the coefficient of heat transfer from the rotating disk (b), with variation of Re and φ: a) 1, 2, 3: values of $\overline{p}$ for $\varphi = -5$, and Re of 1, 10, 50, respectively. (In calculating $\overline{p}$ along the ordinate axis the factor 2/5 was introduced); 4, 5, 6: Re = 10, $-\varphi$ equal to 1, 5, 10, respectively; b) 2, 3, 4: Re = 10; $-\varphi$ equal to 1, 5, 10, respectively; 1, 5: $-\varphi = 5$, Re equal to 1, 50, respectively.

Effect of Reynolds Number Re and Mass Flow Parameter φ on the Coefficient of Resistance Moment c_M and $(Nu)_{av}$

	$\varphi = -1$	$\varphi = -5$	$\varphi = -10$
Re = 10	$20\,c_M = 1.694$ $(Nu)_{av} = 3.80$	1.944 6.64	1.844 8.60
	Re = 1	Re = 10	Re = 50
$\varphi = -5$	$20\,c_M = 15.7$ $(Nu)_{av} = 3.06$	1.94 6.64	0.18 11.9

by Tesla [6]: when a centripetal swirling stream is supplied there is transfer of angular momentum from the flow to the disk. With increase of mass flow (at a fixed Re) the friction coefficient $|\overline{\tau}_\varphi|$ increases (Fig. 3), while with increase of Re (at fixed φ) $|\overline{\tau}_\varphi|$ decreases. Because of the change of sign and the displacement of the point on the disk with $\tau_\varphi = 0$ towards larger radius as the mass flow increases, the moment of resistance increases slightly and may even decrease (see the table), in contrast with the case where material is supplied radially from the center to the periphery, when τ_φ and the moment increase sharply with increase of mass flow (compare [4]). This phenomenon was observed first by Sedach and Nespela [1]. It should be stressed once more that there is a strong effect of radial swirl on the moment, for a linear initial swirl almost the whole of the disk operates in the "turbine" regime. Therefore, in carrying out tests to determine the moment, it is especially necessary to measure the swirl in the initial section.

The radial friction stress component decreases with increase of Reynolds number and increases sharply with increase of mass flow, although $\tau_r : \rho(v_{r_0})^2$ even decreases with increase of $|\varphi|$. The pressure decreases in the flow direction (Fig. 4), at large values of Re and $|\varphi|$, almost in inverse proportion to φ^2. The initial swirl, which has a marked influence on τ_φ, has a weak effect on the pressure gradient and on the radial component τ_r of friction stress.

The nature of the temperature profiles (computation was done for air with Pr = 0.7) changes sharply only in the initial section, and then varies weakly (see Fig. 2). This is reflected correspondingly in a change of the heat transfer coefficient along a radius (Fig. 4). With increase of Re and $|\varphi|$, the local and mean values of heat transfer coefficient increase (Fig. 4 and the table).

The calculations show that the initial swirl has a small effect on heat transfer in conditions where the temperature drop is constant.

The temperature profile has a substantial influence in the initial section.

NOTATION

v_r, v_φ, and v_z are the radial, circular, and transverse components of the velocity vector, respectively; p is the pressure; T is the temperature; ρ is the density; ν is the kinematic viscosity; $\mu = \rho\nu$; ω is the angular velocity; r_0 is the initial radius; s is the gap width; $\overline{\tau}_r = \tau_r : \rho s \omega v_{r_0}$; $\overline{\tau}_\varphi = \tau_\varphi : \rho r_0 s \omega^2$; $\overline{p} = (p - p_0) : \rho(v_{r_0})^2$; $Nu = -(\partial T/\partial z)_d : (T_d - T_s)$; $p_0 = p|_{r=r_0}$; $c_M = M : 2\pi r_0^4 s \rho \omega^2$, where the moment is $M = -2\pi \int_0^{r_0} r^2 \tau_\varphi\, dr$. Subscripts: s refers to the screen, d to the disk.

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